



TITLE:

ABSOLUTELY CONTINUOUS INVARIANT
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CRITICAL AND SINGULAR POINTS
(Dynamical Systems : with Hyperbolicity and
with Large Freedom)

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ABSOLUTELY CONTINUOUS INVARIANT MEASURES FOR CIRCLE MAPS WITH CRITICAL AND SINGULAR POINTS

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Motivated by a global study of the dynamics of periodically perturbed dissipative double homoclinic loops [5], we consider a two-parameter family $(f_{a,L})$ of maps on the circle $S^1 = \mathbb{R}/\mathbb{Z}$ given by

$$f_{a,L}: \theta \mapsto \theta + a + L \log |\Phi(\theta)|, \quad a \in [0, 1), L > 0.$$

The $\Phi: S^1 \rightarrow \mathbb{R}$ is a Morse function, with its graph intersecting the θ -axis transversely. The value of $f_{a,L}$ is undefined at $S = \{\theta: \Phi(\theta) = 0\}$, which is a finite set. All the θ -derivatives blow up to infinity at S . The $f_{a,L}$ has a finite number of critical points.

Main Theorem. [2] *For all large L , there exists a set $A_L^{(\infty)}$ in $[0, 1)$ with positive Lebesgue measure such that for all $a \in A_L^{(\infty)}$, the corresponding $f_{a,L}$ admits a unique absolutely continuous invariant probability measure μ . Lebesgue almost every $\theta \in S^1$ is μ -generic, that is,*

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \varphi(f_{a,L}^i \theta) = \int \varphi d\mu \quad \text{for all continuous } \varphi: S^1 \rightarrow \mathbb{R}.$$

Moreover, the Lebesgue measure of $A_L^{(\infty)}$ satisfies $\lim_{L \rightarrow \infty} \text{Leb}(A_L^{(\infty)}) = 1$.

For the construction of the parameter set $A_L^{(\infty)}$, we perform an inductive parameter exclusion in the spirit of Benedicks and Carleson. To deal with the effect of the singular set, and to get a good estimate of the measure as in the last line of the statement, some additional considerations are necessary. For the construction of the acip, we follow a standard inducing argument. The uniqueness of acip and the genericity follow from the assumption that L is large.